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Can a hazardous event be another source of poverty traps ?

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Abstract

This paper aims to present a new explanation for poverty traps, by the presence of hazardous event probability. We show that adaptation and mitigation policies have different effects on the occurrence of poverty traps : the former could cause a poverty trap while the latter could save from the trap since it decreases the abrupt event probability. As a result, we present a new trade-off between adaptation and mitigation policy other than the usual dynamic trade-off highlighted in many studies (Zemel (2015), Tsur and Zemel (2015)), which is crucial for developing countries. Our simulation results show that a trapped economy adopts an aggressive exploitation policy with higher abrupt event risk while the economy at high equilibrium becomes more precautionary. We also show that when the economy faces a higher risk, the social planner gives more weight to adaptation than to mitigation activity.

Keywords : Abrupt damage, Occurrence Hazard, Multiple Equilibria, Poverty traps, Adaptation, Mitigation.

JEL Classification : O13, D81,Q2, Q54,

1 Introduction

As a result of hazardous climate events which might entail negative consequences, a social planner should consider ways to avoid the damage. A direct response requires action to reduce the probability that a harmful event takes place. In many cases, mitigation activities are able to reduce the risk of an abrupt event by improving environmental quality[1] but can not eliminate it completely. In such situations, a possible action could be alleviating the negative consequences of abrupt damage. The measures taken in this sense to reduce the loss due to abrupt event can be considered as adaptation. The management of adaptation and mitigation activities raises an interesting dynamic trade-off that can be described as "adaptation and mitigation dilemma" in environmental economics literature (Zemel (2015), Tsur and Zemel (2015), Crépin et al. (2012)).

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[1]Since we focus on environmental quality in the paper, mitigation activities are aimed to increase environmental quality. A possible example for mitigation activity can be reforestation, which enhances carbon sinks. (IPCC (2007))

To elaborate more on these concepts, let's take concrete examples. Improvements in energy efficiency, activities such as carbon capture and storage, reforestation address the root causes, by decreasing greenhouse gas emissions and reduce the risk of an abrupt climate event and therefore can be referred to as mitigation. Mitigation activity can also be seen as a tool to avert an abrupt event (Martin and Pindyck (2015)). Whereas, installing flood defenses, developing irrigation systems aim to reduce the inflicted damage of a possible abrupt event and hence can be classified as adaptation. In this context, adaptation plays a *proactive* role, which means that it has no concrete effect prior to abrupt event (Smit et al. (2000) ; Shalizi and Lecocq (2009)). In this example, the problem is to decide for an optimal combination of risk-reducing and damage-reducing measures within a given budget.

However, there exists a second strand of literature on adaptation and mitigation which offers a different definition of adaptation policy. In these models without uncertainty, adaptation helps to decrease the damage from the pollution stock, which affects the utility function rather than the occurrence probability (Bréchet et al. (2012), Kama and Pommeret (2016), Millner and Dietz (2011)).

The aim of this paper is to analyze the effects of environmental policy such as adaptation and mitigation on development traps, also coined poverty traps. The will to offer explanations for development paths of economies through environmental aspects is not that old. Nonetheless, the analysis of poverty of poor countries has ignored environmental aspects for a long time (Dasgupta and Göran-Mäler (1997)). First studies at the intersection of development and environmental economics focused on the relationship between natural resources and the possibility of a sustainable growth path. These studies formed the theoretical basis for "sustainable development" notion (Byrne (2002), Bovenberg and Smulders (1996),Barbier and Homer-Dixon (1999), Schou (2002), Stokey (1998)). Besides, more recent studies concentrated on the direct link between environment and poverty traps by taking wealth distribution (Ikefuji and Horii (2012)), natural resource degradation (Barbier (2010)) and endogenous population dynamics (Constant et al. (2014)) into account.

However, all these studies attempting to shed light on the links between environmental issues and economic development don't provide any insight either on the role of abrupt event possibility or on possible adverse effects of environmental policy for occurrence of poverty traps. We think it is important to focus on the overlooked link between hazardous events and poverty traps since abrupt climate events are one of the major concerns in developing countries (IPCC (2007), Mechler (2010)).

In order to fill this gap in literature, we construct a simple model that incorporates adaptation and mitigation policies in an economy subject to hazardous event probability. First, endogenous abrupt event probability is shown to be likely to cause a poverty trap (i.e multiple equilibria). Additionally, we show that adaptation policy can cause multiple equilibria while mitigation can avoid it and this depends on the occurrence probability. Indeed, this is not to say that social planner should not implement adaptation policy. Then, a legitimate question to be addressed is : how can a policy maker implement adaptation policy without causing a poverty trap ? Our response to this question is simply that she should implement adaptation policy coupled with mitigation policy. We will further discuss the mechanisms behind this normative recommendation in the remainder of the paper.

The main results and mechanisms of this paper can be summarized as follows : The reason why an abrupt event possibility may cause a poverty trap is that, when an economy faces an abrupt event probability, a second trade-off arises between consumption and hazardous event occurrence other than the usual intertemporal trade-off between present and future consumption. Basically, agents in a region with serious environmental quality problems are supposed to be more impatient due to endogenous hazard rate. Therefore, agents tend to increase their consumption at earlier dates since they face a higher event probability, which again stresses environmental quality over time. This trade-off between consumption and abrupt event results in a vicious cycle of "low level of environmental quality and consumption" at long run, which can be defined as a poverty trap. However, agents in a region with high level of environmental quality can fix farsighted goals since the occurrence probability is relatively lower. Thus, they would benefit from a high level of environmental quality and consumption in the long run.

What about the role of environmental policy on poverty traps ? We show that adaptation policy can cause multiple equilibria while mitigation can avoid it, and this depends closely on the occurrence probability. The reason behind is the following : adaptation capital is shown to decrease environmental quality level as agents can worry less about the consequences of an abrupt event with an increasing adaptation capacity, a similar result to Zemel (2015) where the pollution stock increases with adaptation capital. Then, since abrupt event probability increases, the trade-off between present consumption and hazardous event becomes tighter, which is likely to raise multiple equilibra. Contrary to this mechanism, mitigation activity improves the environmental quality and the trade-off between present consumption and hazardous event turns to be weaker. In our model, there exists always a risk of collapse whatever the environmental quality level is. However, to understand this result, assume for a moment that mitigation activity can eliminate the collapse risk and in this case, the trade-off between present consumption and hazardous event disappears since there is no risk of collapse. Consequently, the poverty trap is not a possible outcome.

Numerical simulations confirm as well the mechanisms explained above by showing that a trapped economy facing a higher collapse risk adopts an aggressive exploitation of natural resources (i.e environmental quality). However, an economy at the high equilibrium becomes more precautionary with higher risk. This result is merely in contrast with Tsur and Zemel (2016) and Bommier et al. (2015) where authors show that hazardous event probability makes the economy always precautionary. We also show that when environmental policy consists of only adaptation activity, the economy at lower equilibrium accumulates more adaptation capital than the economy at high equilibrium. Nonetheless, in the economy implementing both policies, ratio of adaptation and mitigation increases with higher risk since marginal benefit from adaptation capital increases. Besides, since the economy implements also mitigation policy, the long run level of environmental quality may be higher despite adaptation policy.

The remainder of the paper is organized as follows. The section 2 presents the benchmark model with necessary conditions to have multiple equilibria economy. Section 3 explains the model with adaptation and mitigation policy and explains how the former could cause a poverty trap while the latter could avoid it. Section 4 presents the numerical simulation results and last section 5 concludes the paper.

2 Model

Let $S(t)$ represent environmental stock available or environmental quality at time t. e.g, the stock of clean water, soil quality, air quality, forests, biomass. We refer to a broad definition for environmental quality which encompasses all environmental amenities and existing natural capital that have an economic value[2]. Obviously, disamenities such as waste and pollution stemming from consumption decrease environmental quality stock. The stock $S(t)$ evolves in time according to

$$\dot{S}(t) = R(S(t)) - c(t) \tag{1}$$

[2]We exclude mining and oil industry from our definition of natural capital.

where the control variable $c(t)$ stands for consumption at time t. With a given initial state $S(0)$, an exploitation policy of environmental stock $c(t)$ generates the state process $S(t)$ according to equation (1) and provides the utility $u(c(t), S(t))$. Similar to Kama and Schubert (2007), we use a framework where consumption comes directly from environmental services and causes environmental damages.

We make use of the following assumptions.

A.1 The regeneration of environmental is characterized by $R(.) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, $R(S) > 0$, $R^{'}(S) > 0$ and $R^{''}(S) < 0$.

A.2 The utility function $u(.) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable with following properties ; $u(c) < 0$, $u^{'}(c) > 0$, $u^{''}(c) < 0$, $\forall c$ and $\lim_{c\rightarrow 0} u^{'}(c) = \infty$.

In addition to the fact that environmental stock $S(t)$ represents a source of revenue for economy, it also affects the occurrence probability of a catastrophic event. The abrupt event is described by the occurrence probability and results in a regime with an inflicted environmental damage. The consequences of this regime are defined by the post-event value $\varphi(S)$ that we will discuss later.

Similar to Tsur and Zemel (2016), let T the event occurrence time and denote $F(t) = Pr\{T \leq t\}$ and $f(t) = F^{'}(t)$ as the corresponding probability distribution and density functions respectively. The environmental stock dependent hazard rate $h(S)$ is related to $F(t)$ and $f(t)$ with respect to

$$h(S(t))\Delta = \frac{f(t)\Delta}{1 - F(t)} = -\frac{d[ln(1 - F(t))]}{dt} \tag{2}$$

where Δ is an infinitesimal time interval. The term $h(S(t))\Delta$ specifies the conditional probability that an abrupt event will occur between $[t, t + \Delta]$ given that event has not occurred by time t. A formal specification for probability distribution and density functions gives

$$F(t) = 1 - exp\left(-\int_0^t h(S(\tau))\,d\tau\right) \;\; and \;\; f(t) = h(S(t))[1 - F(t)] \tag{3}$$

Since $S(t)$ is a beneficial state, hazard rate h is a non-increasing function. Given the uncertain arrival time T, the exploitation policy $c(t)$ yields the following payoff

$$\int_0^T u(c(t))\,e^{-\rho t}dt + \varphi(S(T))\,e^{-\rho T} \tag{4}$$

where ρ is the social discount rate. Taking expectations of the expression (4) according to distribution of T and considering (3) gives the expected payoff

$$\int_0^\infty U(c(t), S(t))\,exp\left(-\int_0^t [\rho + h(S(\tau))]\,d\tau\right)dt \tag{5}$$

where

$$U(c(t), S(t)) = u(c(t)) + h(S(t))\,\varphi(S(t)) \tag{6}$$

is the instantaneous utility including an abrupt event threat. Similar to Tsur and Zemel (2016), we consider a single-occurrence event which entails an immediate damage ψ for the sake of analytical simplicity. This type of event with irreversible negative consequences is usually considered as a "doomsday" event.[3] An example of this kind of irreversible event can be the massive intrusion of saline into freshwater stock, which

[3]see Tsur and Zemel (2006) for a detailed discussion.

becomes impossible to be recovered completely afterwards. The post-value function describing economy after the occurrence of catastrophic event is defined as

$$\varphi(S) = \int_0^\infty u(c_{min}) e^{-\rho t} dt - \bar{\psi} = -\bar{\psi} \tag{7}$$

where the consumption level is reduced to c_{min} by policymaker to not fall below the level $S(T)$ after occurrence. Note that this subsistence level of consumption does not provide utility to agents. The specification of post-regime function can be considered as a restrictive one. However, the use of a more complicated post-value function with multiple occurrence events does not change the main mechanisms of the paper but would yield tedious calculations.

The solution of maximizing (5) with respect to evolution of environmental stock (1) leads to Keynes-Ramsey rule (8), see Appendix (6.1) for details,

$$\dot{c} = -\frac{u'(c)}{u''(c)} \left[R'(S) - \theta(S) - \frac{\bar{\psi} h'(S)}{u'(c)} - \frac{\theta'(S)}{\theta(S)} \left[\frac{u(c) - \bar{\psi} h(S)}{u'(c)} + \dot{S} \right] \right] \tag{8}$$

where $\theta(S) = \rho + h(S)$ for the sake of notation simplicity. For $\theta(S) \to \bar{\theta}$, the result reduces to usual Keynes-Ramsey rule of the standard neoclassical growth model. Compared to standard growth model, we have two additional effects with an endogenous hazard rate (Strulik (2012)) : growth and level effect. The growth effect (last term in (8)) is unambiguously positive on consumption growth. Protecting the environment (a higher environmental quality stock) makes people more patient since abrupt event probability decreases with higher environmental stock. It follows that people start to fix far-sighted goals. Consequently, they tend to accumulate more natural resources rather than depleting them quickly. (i.e higher $\dot{c}/c$)

The level effect is more interesting. On the one hand, since a catastrophic event is undesirable for welfare, society becomes more precautionary through the term $-\bar{\psi} h'(S)/u'(c)$. On the other hand, a higher hazard rate entails a lower economic growth and a higher adjusted discount rate $\theta(S)$. This level effect is reflected by the term $\left(\theta'(S)/\theta(S)\right)\left(\left(u(c) - \bar{\psi} h(S)\right)/u'(c)\right)$ and has a negative effect on growth since agents value the future less than today.

If the society becomes precautionary through the level effect, we can say that economy becomes precautionary by both level and growth effect channels and does not face a poverty trap. However, in case where agents become aggressive to exploit natural capital from the level effect which could dominate the growth effect, then economy is likely to face multiple equilibria. [4]

Proposition 1. *Multiple steady state (i.e poverty trap) possibility arises if only economy is exposed to occurrence hazard depending on environmental stock S. The necessary condition to have a poverty trap is given by $\exists S < \bar{S}$ such that ;*

$$G'(S) = R''(S) - \theta'(S) - \left[\frac{\theta''(S)\theta(S) - \left(\theta'(S)\right)^2}{(\theta(S))^2} \right] \frac{\left(u(c) - \bar{\psi} h(S)\right)}{u'(c)} - \frac{\bar{\psi} h''(S)}{u'(R(S))}$$

$$+ \frac{\theta'(S)}{\theta(S)} \frac{\bar{\psi} h'(S)}{u'(R(S))} - \left[\frac{\theta'(S)}{\theta(S)} \left(1 - \frac{\left(u(c) - \bar{\psi} h(S)\right) u''(c)}{\left(u'(c)\right)^2} \right) - \frac{\bar{\psi} u''(R(S)) h'(S)}{\left(u'(c)\right)^2} \right] \left[R'(S) \right] > 0 \tag{9}$$

[4]This mechanism is also supported by the numerical analysis in the remainder of the paper.

Proof. See Appendix (6.2)

In a standard neoclassical growth model, this condition cannot be satisfied since all terms with endogenous abrupt event probability vanish and the condition reduces to $R^{''}(S) > 0$, which is not possible according to A.1. In a Ramsey-Cass-Koopmans model, the usual inter-temporal trade-off is between present and future consumption. Moreover, an economy exposed to abrupt events faces an additional trade-off between present consumption and catastrophic risk. Indeed, this is the reason why the economy could find itself in a trapped equilibrium.

It would be preferable to find an analytical threshold for environmental quality level under which a poverty trap appears. However, finding this kind of threshold is even difficult for less complicated models (Grass (2008), chapter 5.) For the sake of space, we don't focus on this issue within this paper.

Before explaining more in depth the economic intuition behind the occurrence of poverty traps, more concretely, one can understand the occurrence of poverty traps due to hazard rate by making a phase diagram analysis. Recall that the consumption rule is $\dot{c}/c = \sigma(r - \rho)$[5] without catastrophic event probability. Then, the steady-state curve $\dot{c} = 0$ is vertical and implies a unique equilibrium, which is not the case with hazard probability. Considering equation (8), we can remark that steady state curve $\dot{c} = 0$ is non-linear on a phase plane (S, c). Therefore, multiple equilibria is a possible outcome with event risk. To illustrate this explanation, we refer to a phase diagram analysis in the following section.

2.1 Phase Diagram Analysis

Finding directions of arrows on the phase diagram analysis requires some attention since steady state curve of consumption is a function of environmental quality S. If (c, S) is below (above) the $\dot{S} = 0$, $R(S) - c > (<)\, 0$, which makes $\dot{S} > (<)\, 0$. The analysis is not that easy for $\dot{c} = 0$. Therefore, we use the necessary condition (9). Above the $\dot{c} = 0$ line, we have $\dot{c} > (<)\, 0$ if $G^{'}(S) > (<)\, 0$. We precise different zones on the phase diagram where the slope of $G(S)$ changes. The possible phase diagrams of the dynamical system are as follows ;

[5]where $r = R^{'}(S)$. With usual growth model, one can maximize $\int_0^\infty \frac{c^{1-\sigma}}{1-\sigma} e^{-\rho t} dt$ with respect to $\dot{S} = R(S) - c$ and find the usual Euler equation.

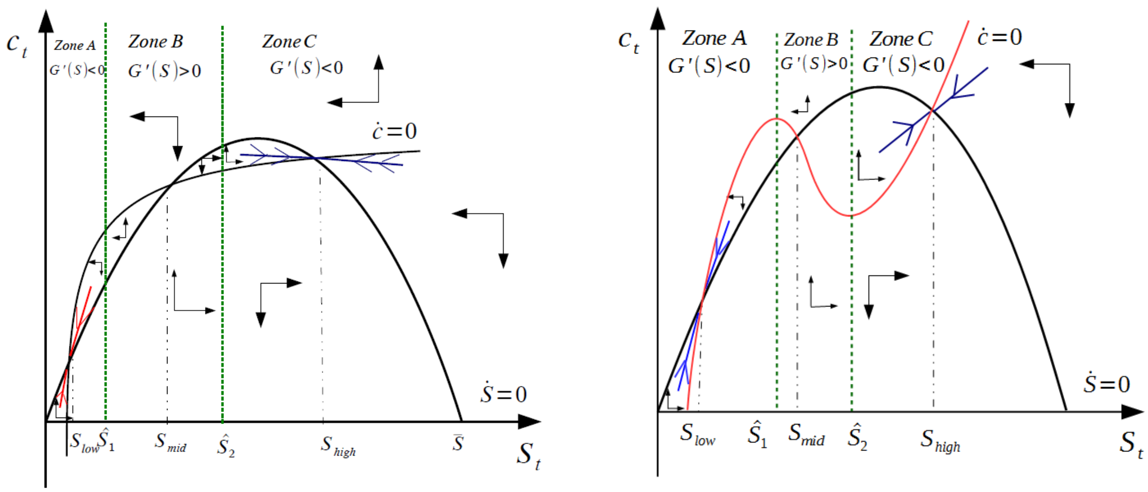

Figure 1: Phase diagram with monotonically and non-monotonically increasing $\dot{c} = 0$ curve

Note that, from multiple steady state condition, between $\left[0; \hat{S}_1\right]$ and $\left[\hat{S}_2; \bar{S}\right]$, we know that $G^{'}(S) < 0$ (see Figure (10)). Within the zones A and C, the slope of the $\dot{c} = 0$ line is always positive, which means that the curve is always increasing between these intervals (see Appendix *(6.2)* for details). Conversely, within the zone B, it is not possible to determine exactly whether the slope of $\dot{c} = 0$ line is increasing or decreasing. Consequently, we rigorously present two different phase diagrams.

Direction of arrows in Figure (1) shows that there exists three steady-state with one being unstable and two others being stable. Furthermore, subsequent analysis of stability of dynamical system will stipulate that there could be complex dynamics around the middle steady-state.

Lemma 1. *The steady-states* (S_{low}, c_{low}) *and* (S_{high}, c_{high}) *are saddle path stable. However,* (S_{mid}, c_{mid}) *could have complex dynamics.*

Proof. See Appendix (6.4)

This means that the economy would converge to either high or low equilibrium. Once the economy reaches equilibrium (low or high), it stays definitely there. The economy reaching the low equilibrium is said to be "trapped" where the consumption and environmental quality are lower relative to high equilibrium.

In order to deepen our understanding about the multiplicity of equilibria, we reformulate equation (8) (Schumacher (2009))

$$\dot{c} = -\frac{u^{'}(c(t))}{u^{''}(c(t))}\left[R^{'}(S(t)) - \theta(S(t)) - \frac{\bar{\psi}h^{'}(S(t))}{u^{'}(c(t))} - \frac{\theta^{'}(S(t))}{u^{'}(c(t))}\int_t^{\infty}\left(u(c(\tau)) - \bar{\psi}h(S(\tau))\right)exp(-z(\tau))\,d\tau\right] \tag{10}$$

where $z(\tau) = \int_0^t \theta(S(\tau))\,d\tau$. The multiple steady-state takes place when an economy is very poor. Then, postponing consumption is too costly for survival ($u^{'}(0) = \infty$) and preferences are directed toward today. In this case, agents are exploiting most of natural capital at earlier dates and they face a higher event probability. Since the occurrence probability is high, agents could tend to be impatient[6] and start to exploit

[6]This result cannot always be true since we have shown two different level effects due to occurrence probability. (see equation

excessively natural resources. Consequently, a vicious cycle could occur due to the trade-off between present consumption (i.e the use of environment) and hazard rate and trap an economy to a poverty trap with lower environmental quality and consumption level.

More formally, a higher marginal utility decreases the importance of last two terms in equation (10). This implies a lower impact of occurrence probability on trade-off between present and future consumption. Therefore, developing countries are expected to be less sensitive to environmental stock changes since they aim to fulfill basic needs. Indeed, Environmental Performance Index in 2016[7] supports this result since many African and Asian countries are listed at bottom of the ranking. A striking real world example can be the deforestation trend in Asian countries. Margano et al. (2014) show that there was a loss of %40 of total national forests in Indonesia between the period 2000-2012.

Contrary to this explanation, an economy protecting the environment is exposed to a lower hazard rate. Agents are more patient and have far-sighted goals since the mortality rate is expected to be lower.

Low environmental quality and overuse of environmental assets[8] represent important environmental concerns in many African and Asian countries (Environmental Outlook to 2030, OECD (2008)). Our theoretical model shows that hazardous event probability may be an important factor to understand why some countries are (or would be in future) trapped to lower environmental quality and consumption level. One may say that occurrence probability could trigger itself abrupt events and cause also poverty traps since it raises the impatience level of society and exposes the economy to a trade-off between present consumption and abrupt event.

At first glance, the link between endogenous hazard rate and poverty traps may be counter-intuitive. This is because in many studies abrupt event probability depends mostly on global pollution stock rather than a general variable such as environmental quality stock (Zemel (2015), Tsur and Zemel (2015), Bommier et al. (2015)). Our paper differs also in many sense from Zemel (2015), Tsur and Zemel (2015) which does not study the link between abrupt events, environmental policy and poverty traps. In this regard, by concentrating on these aspects, we believe we give important intuitions to policy makers and provide a new perspective to adaptation/mitigation dilemma which is a hot topic in environmental policy discussions.

In this paper, differently from literature, we also argue that the use of environment in a given region affects locally the abrupt event risk. Therefore, each region may end up with different levels of consumption and environmental quality stock.

To elaborate this more in depth, for example, forests influence local and regional climate by creating micro-climate that affects the existing ecosystem in a given area. (Dasgupta and Göran-Mäler (1997), chapter 1) and helps to decrease a possibility of an abrupt event. (Jie-Sheng et al. (2014), Bradshaw et al. (2007)) An interesting real world example could be the reforestation project in Samboja Lestari conducted by Borneo Orangutan Survival Foundation. Project helped to increase rainfall by 25% and to lower air temperature by 3° to 5°. (Boer (2010), Normile (2009))

With all these elements, one may understand how local environmental conditions through abrupt event probability could cause a poverty trap in a country. Indeed, since the use of environment is one of the major source of revenue in developing countries,[9] countries with low initial state of environmental quality are likely

(8)) However, one may say that low-income countries would be impatient (see variable $\theta(s)$) since the amplitude of two level effects decreases with higher marginal utility of consumption. Then, growth rate of the economy will be lower.

[7]The Environmental Performance Index is a method developed by Yale Center for Environmental Law and Policy, that evaluates environmental policies of countries. see http://epi.yale.edu/country-rankings.

[8]Some examples could be permanent like clean water stress, decreased soil quality and lack of clean water stocks.

[9]see http://data.worldbank.org/indicator/NY.GDP.FRST.RT.ZS?name_desc=false&view=map for a detailed data on forests rents.

to suffer from poverty trap due to the mechanism explained above.

3 Model with Environmental Policy

In this section, we consider the benchmark model (5) - (1) and analyze how adaptation and mitigation policies shape main results obtained in the model without environmental policy. Especially, our focus will be on the implications of adaptation and mitigation policies on poverty traps.

In our model, as mentioned in the benchmark model, when an abrupt event occurs, economy suffers from environmental damage. However, a social planner could reduce the damage ψ via adaptation capital K_A. This kind of modeling adaptation in the same line with Zemel (2015) andTsur and Zemel (2016) differs from Bréchet et al. (2012) and Kama and Pommeret (2016) where the adaptation capital affects directly the damage function for all time t. In our specification, as mentioned above, adaptation plays a *proactive* role, which means that concrete benefits of adaptation can be gained if only an abrupt event occurs. However, this is not to say that investing in adaptation decision makes no difference. Its contribution is accounted for by the objective function of the social planner. Investing at rate A contributes to adaptation capital K_A which follows the stock dynamics

$$\dot{K}_A(t) = A(t) - \delta K_A(t) \tag{11}$$

where δ represents the capital depreciation rate and damage function $\psi(K_A)$ decreases when adaptation capital K_A increases. We assume that

A.3 The damage function is characterized by $\psi(.)$: $\mathbb{R}_+ \to \mathbb{R}_+$, $\psi(0) = \bar{\psi}$, $\psi(\infty) = \underline{\psi}$, $\psi(K_A) > 0$, $\psi'(K_A) < 0$ and $\psi''(K_A) > 0$

When there is no adaptation expenditure, the inflicted penalty will be a constant term as in Tsur and Zemel (2016). Moreover, it is realistic to assume that reduction in damage has a limit since we cannot get rid of completely from negative effects of a catastrophic event by accumulating adaptation capital. For that reason, we assume that penalty function is constrained between an upper and lower bound.

Investing at rate M for mitigation that improves the environmental quality. Then, with the presence of mitigation activity, environmental quality evolves according to

$$\dot{S}(t) = R(S(t)) - c(t) + \Gamma(M(t)) \tag{12}$$

where $\Gamma(M)$ holds for mitigation that encompasses all activities such as reforestation, desalination of water stock, enhancing carbon sinks etc. Mitigation is defined as a "human intervention to reduce the sources or enhance the sinks of greenhouse gases." (IPCC (2014), p.4) In this sense, reforestation can be considered as a means to enhance carbon sinks since forests allow for carbon sequestration.

The specification for mitigation variable is in the same line with Chimeli and Braden (2005). Alternatively, function $\Gamma(M)$ can be considered as "environmental protection function". The expenditures for environmental protection may be directed not only toward pollution mitigation but also toward protection of forests, recovery of degraded areas. Equivalently, mitigation activity can be seen as a means of improving environmental quality.

In order to keep the model as simple as possible, we choose to consider mitigation as a flow variable. We make the use of following assumption

A.4 The mitigation function is given by $\Gamma(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with following properties ; $\Gamma(M) > 0$, $\Gamma'(M) > 0$, $\Gamma''(M) < 0$ and $\Gamma(0) = 0$.

The mitigation function is assumed to be an increasing and concave function. Note that mitigation activity can be considered as a complement to regeneration of environment. In an economy with adaptation and mitigation activities, the expected payoff is

$$\int_0^\infty U(c(t), A(t), M(t), S(t)) \, exp\left(-\int_0^t [\rho + h(S(\tau))] \, d\tau\right) dt \tag{13}$$

where the instantaneous utility including catastrophic threat is

$$U(c(t), A(t), M(t), S(t)) = u(c(t), A(t), M(t)) + h(S(t)) \varphi(S(t)) \tag{14}$$

The instantaneous utility increases with consumption and decreases with adaptation and mitigation expenditures $A(t)$ and $M(t)$ as in Zemel (2015) and Tsur and Zemel (2016)[10] (since expenditures for adaptation and mitigation come at the expense of consumption.[11])

A.5 The utility function is given by $u(.) : \mathbb{R}_+ \to \mathbb{R}_+$ is twice continuously differentiable with following properties ; $u_c(c, A, M) > 0$, $u_M(c, A, M) < 0$, $u_{MM}(c, A, M) = 0$, $u_A(c, A, M) < 0$ and $u_{AA}(c, A, M) > 0$.

The optimal policy is to maximize (13) subject to (12) and (11) (see details in Appendix (6.5)). Essentially, the control variables $c(.)$, $M(.)$ and $A(.)$ are determined by shadow prices $\lambda(.)$ and $\mu(.)$ corresponding to S and K_A. Explicit expressions for dynamics of shadow prices are given in Appendix (6.5)[12].

Since the focus of our study is to figure out the effect of adaptation and mitigation policies on poverty traps, we need to know how steady-state level of adaptation capital K_A and mitigation activity M change with respect to environmental quality stock S. In more technical terms, one should know the effect of these two central policies on necessary condition (9) for multiple equilibria.

A steady-state of the optimization program (13) is a couple (K_A, S) approached by an optimal process $(K_A(.), S(.))$. Once the process reaches steady-state, the hazard rate $h(S)$ becomes constant and behaves as another component of the discount factor. As stated in Zemel (2015), the problem at hand is a deterministic one and the objective function is also deterministic, yielding a value that corresponds to maximum expected value of the uncertainty problem. If the process enters a stationary state, it would stay at point (K_A, S) indefinitely, without being disturbed.

Occurrence of catastrophic event causes a damage but otherwise social planner keeps the same optimal steady state policy prior to occurrence. This is not to say that inflicted penalty does not have any effect on decision making of social planner. The negative consequences of a possible catastrophic event are already taken into account in the objective function (13).

[10]By putting the cost of adaptation and mitigation in the resource constraint of the dynamic problem, one can find same results. However, this yields more tedious calculations.

[11]By looking at equation (37). (Appendix (6.5)). it is easy to see that higher consumption implies lower mitigation at optimum and vice versa.

[12]Another possible and probably more easier way to present our results would be to use L-method of Tsur and Zemel (2015) for multi-state control problems. However, this method allows an analysis only on long term properties and not on transitional dynamics. Since, we conduct a numerical analysis to justify the main mechanisms of the model, we choose to use standard optimal control methods to solve the problem at hand.

Consider the loci for possible steady states in the (K_A, S) plane. We set $\dot{A} = 0$ in (39)(see Appendix (6.5)) and by using $A = \delta K_A$ at steady-state to have

$$Q_1^{'}(\delta K_A)(\theta(S) + \delta) = -\psi^{'}(K_A)\, h(S) \tag{15}$$

which defines the function $S(K_A)$. The graph of $S(.)$ represents a curve on (K_A, S) plane, denoted the steady-state curve and having the following economic intuition : right-hand side can be interpreted as the marginal benefit of adaptation capital and left-hand sight is the marginal cost of adaptation capital. Optimal steady states are located on this curve which takes the slope

$$\frac{dK_A}{dS} = \frac{h^{'}(S)\, Q_1^{'}(\delta K_A)\left(\frac{\rho+\delta}{h(S)}\right)}{(\theta(S) + \delta)\, Q_1^{''}(\delta K_A) + \psi^{''}(K_A)\, h(S)} < 0 \tag{16}$$

The negative slope indicates that when environmental quality is higher, the economy needs less adaptation capital, which case is plausible since the probability of catastrophic event is lower. Equivalently, one may say that when economy accumulates more adaptation capital, natural resources start to be overused.[13] The trade-off between adaptation and mitigation is evident in (16). Making mitigation increases the environmental quality. It follows that occurrence hazard decreases, which case pushes the economy to accumulate less adaptation capital. We can also explain this result by looking at catastrophic treat factor in (13). When environmental quality increases, the weight of this component decreases due to lower hazard rate, which means that there is less incentive to accumulate adaptation capital.

Another interpretation of this result can be the following : since agents expect to face less damage with adaptation policy and can bear more easily the negative consequences of an abrupt event, they tend to care less for the environmental quality.

Following the same type of analysis in order to see the relationship between environmental quality and mitigation activity. We use the first-order condition (37) $u^{'}(R(S) + \Gamma(M))\,\Gamma^{'}(M) = Q_2^{'}(M)$. To facilitate calculations, we suppose a linear cost function for mitigation activity. $Q_2(M) = P_M M$ where P_M is the unit price for mitigation. Optimal steady states should satisfy (37). To see how the steady-state level of mitigation activity M changes with respect to environmental quality S, we calculate

$$\frac{dM}{dS} = -\frac{u^{''}(c)\, R^{'}(S)\, \Gamma^{'}(M)}{u^{''}(c)\,(\Gamma^{'}(M))^2 + u^{'}(c)\,\Gamma^{''}(M)} < 0 \tag{17}$$

The equation (17) will be important to see the effect of mitigation activity on occurrence of poverty traps. It follows that when environmental quality is higher, economy makes less mitigation. In other words, the economy needs less mitigation if environmental quality is higher. To sum up, from (16) and (17), one may observe that when environmental quality is higher, economy chooses to invest less in adaptation and mitigation.

3.1 How environmental policy can cause/avoid a poverty trap ?

In this section, we highlight the mechanisms to understand in which conditions environmental policy could cause or avoid a poverty trap. To facilitate the understanding of the mechanisms, we prefer to analyze separately the effect of adaptation and mitigation policies on poverty traps. We also provide some numerical examples to justify our results. ((6.5.1) and (6.5.2).)

[13] see Zemel (2015) for a similar result.

Case 1. An economy implementing only adaptation policy

We argue that adaptation policy might trap an economy to a low equilibrium. The mechanism is the following : when a policy maker starts to invest in adaptation capital, the environmental quality decreases as shown in (16) and hazard rate increases. Since the preferences of low-income countries are directed towards today[14], an increase in hazard rate due to adaptation capital accumulation amplifies the impatience of low-income countries. Then, these countries become aggressive to exploit more natural resources. It follows that event risk amplifies again with an increasing need for adaptation capital[15]. This mechanism, yielding a vicious cycle, explains why a developing country with high level of marginal utility traps to a low steady-state equilibrium by investing only in adaptation capital. In order to assess the effect of adaptation capital on poverty trap, we calculate the necessary condition for multiple equilibria (see Appendix (6.5.1) for details)

$$G_1^{'}(S) = X_1(S) + \underbrace{\left[\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S) - \theta(S)) + \delta Q_1^{'}(\delta K_A)\right]\right]\frac{dK_A}{dS}}_{=Z_1>0} > 0 \tag{18}$$

The term Z_1 stands for the effect of adaptation capital on poverty trap. Since this term is positive, we can say that the possibility of multiple equilibria increases with adaptation capital. The trade-off between present consumption and catastrophic event becomes more significant and puzzling with adaptation policy since abrupt event risk increases.

Consider a benchmark case where the economy without environmental policy admits an unique equilibrium. Then, a social planner takes the benchmark economy and starts to invest in adaptation capital. As stated above, the possibility of poverty trap increases.

The following numerical example shows this possibility[16] ;

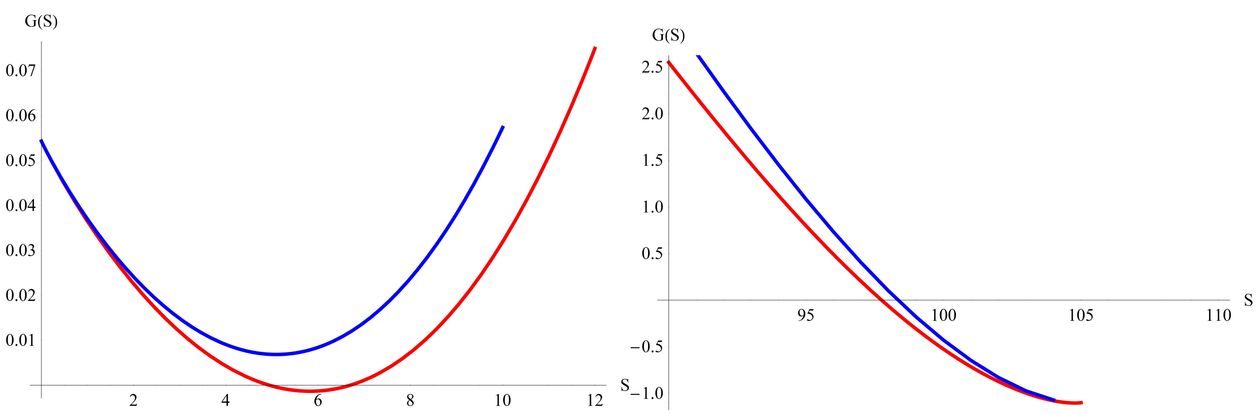

Figure 2: $G(S)$ function (Red and blue curves hold for model with only adaptation policy and without policy respectively.)

We can remark that with adaptation policy, when the economy suffers from poverty trap, the high steady-state level of environmental quality is lower than benchmark unique steady-state level. This means that adaptation policy decreases the steady-state level of environmental quality even for wealthier countries.

[14]Recall from equation (10) that the multiple equilibria occurs when agents are poor and not willing to postpone their consumption. Then, implementing adaptation policy increases hazard rate.

[15]Since the abrupt event risk increases, marginal value of adaptation capital increases.

[16]For the sake of graphical clarity, we split the figure in two parts.

Indeed, a policy recommendation based on more adaptation capital in developing countries could trap these countries to lower equilibrium. This is not to say that social planner should stop to invest in adaptation capital. Rather, she should be aware of eventual adverse effects of adaptation policy and should implement mitigation policy to avoid negative effects of adaptation policy. We analyze the effect of mitigation policy in the following subsection.

Case 1. An economy implementing only mitigation policy

We show that an economy implementing mitigation policy could escape a poverty trap. The reason is the following : since the consumption comes from environmental assets, improving environmental quality (mitigation) increases the consumption level at long run. Therefore, low income countries could have a lower level of marginal utility of consumption, implying that they could start to make far-sighted decisions. As abrupt event probability decreases with mitigation policy, agents will be more patient and and willing to postpone their consumption to the future. In order to see this mechanism in a formal way, we provide the necessary condition for multiple equilibria (Appendix (6.5.2) for details)

$$G_2^{'}(S) = X_2(S) \underbrace{-\frac{\theta^{'}(S)}{\theta(S)}\left[\left(\frac{u^{''}(c)\left(u(c)-\bar{\psi}h(S)-P_M M\right)}{\left(u^{'}(c)\right)^2}\right)-\frac{\bar{\psi}u^{''}(c)\,h^{'}(S)}{\left(u^{'}(c)\right)^2}-\frac{P_M}{u^{'}(c)\,\Gamma^{'}(M)}\right]\Gamma^{'}(M(S))\frac{dM}{dS}}_{=Z_2<0} > 0 \tag{19}$$

Mitigation activity makes less possible that necessary condition for multiple equilibria holds since Z_2 is a negative term. (Appendix (6.5.2) for details.) In our model, mitigation activity can decrease the probability of an abrupt event but cannot totally eliminate it. Indeed, this provides a justification to invest in a proactive adaptation capital.

However, suppose for a moment that mitigation activity can reduce the hazard rate to zero. It follows that the trade-off between present consumption and catastrophic event, which causes multiple equilibria, disappears completely. Therefore, a multiple equilibria is not a possible outcome. Based on these elements, one can understand that mitigation activity lead to a weaken trade-off between present consumption and catastrophic event.

Consider an economy suffering from poverty trap without environmental policy. Then, social planner takes this multiple equilibria economy and starts to make mitigation. We show numerically that economy can escape a poverty trap by mitigation activity.

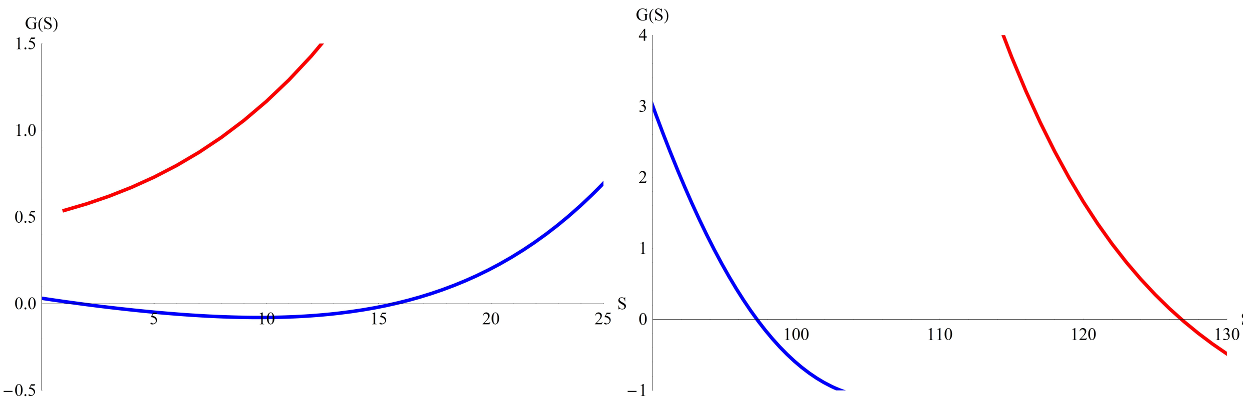

Figure 3: $G(S)$ function (Red and blue curves hold for model with only mitigation policy and without policy respectively.)

Obviously, mitigation activity not only saves the economy from poverty trap but also increases the steady-state level of environmental quality as expected. Indeed, mitigation policy allows the economy to have a higher consumption level. Hence, low-income countries could postpone their consumption to future as they can fulfill more easily basic needs for survival. As a nutshell, a mitigation policy could break the vicious cycle that can be triggered by adaptation policy. Then, one can conclude that social planner must couple the adaptation policy with mitigation policy in order to avoid an eventual poverty trap.

4 Numerical analysis

This section illustrates theoretical findings we obtained in the previous sections. The aim is to examine how the economy reacts to higher catastrophic event probability when it is located at low and high equilibrium. We also present optimal transitional dynamics in the economy implementing both adaptation and mitigation policy and analyze how the economy adjusts its optimal adaptation and mitigation decisions.

The endogenous hazard risk of catastrophic event is represented by the following function similar to Ren and Polasky (2014):

$$h(S) = \frac{2\bar{h}}{1 + exp\left[\eta\left(s/s^{*} - 1\right)\right]} \tag{20}$$

where $\bar{h}$ which is the upper bound for the hazard rate. When $\eta = 0$, the risk becomes exogenous and endogenous when $\eta > 0$. We choose $\bar{h} = 0.5$ and $\bar{h} = 0.25$ for low and high risk profile respectively. The hazard rate takes the form ;

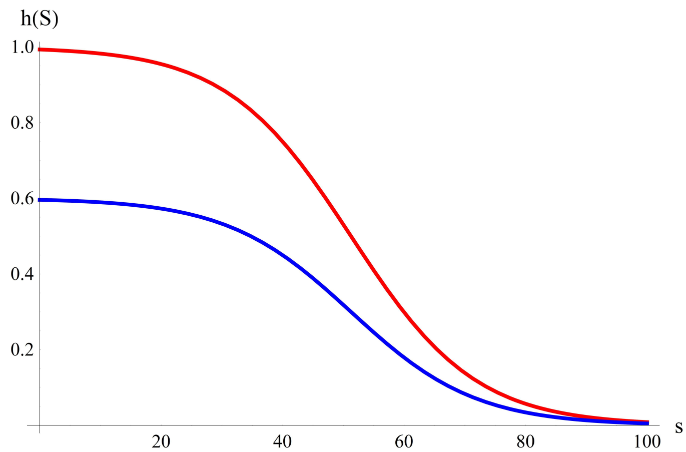

Figure 4: Hazard rate for catastrophic risk (Red and blue curves for high and low risk profile respectively.)

4.1 Benchmark Economy

We investigate how the economy without environmental policy at low and high equilibrium reacts to higher risk profile environment in Figure (6). At low equilibrium, agents become more aggressive and exploit more natural capital. However, at high equilibrium, society becomes more precautionary concerning the use of the environment.

We remark that the economy at poverty trap reacts aggressively to higher occurrence hazard, implying an increase of natural resource exploitation. However, the economy at high equilibrium becomes more conservative for environment. This result differs from Tsur and Zemel (2016),Bommier et al. (2015) and Ren and Polasky (2014)[17] where authors argue that collapse risk makes the society always more precautionary. Contrary to previous studies, our results show that aggressive/precautionary management depends on whether the economy suffers from a poverty trap or not.

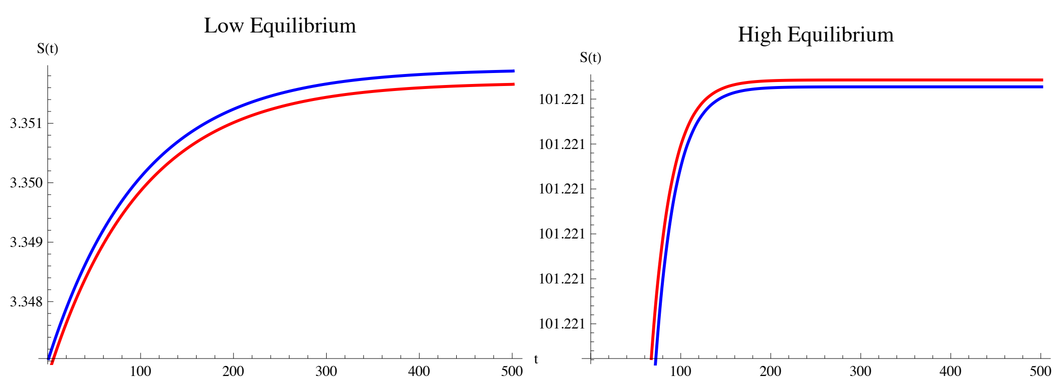

Figure 5: Environmental quality dynamics at low and high equilibrium

[17]Ren and Polasky (2014) differs slightly from Tsur and Zemel (2016) and Bommier et al. (2015), by showing that an aggressive management policy takes place if risk endogeneity is small enough.

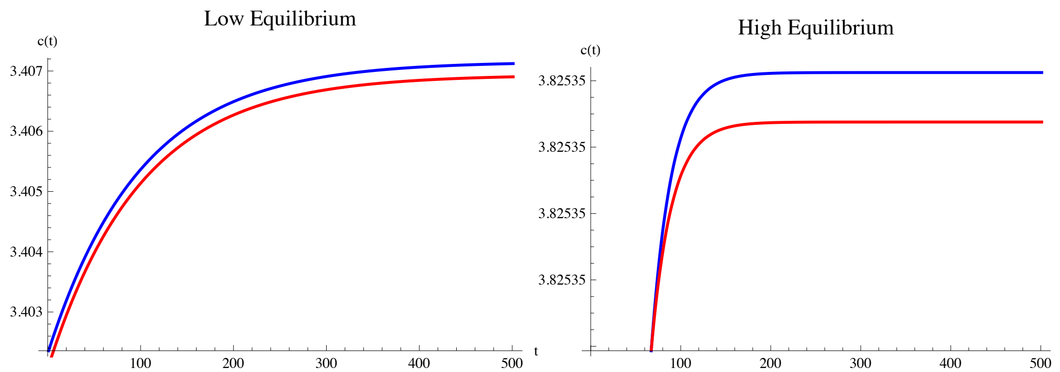

Figure 6: Consumption dynamics at low and high equilibrium

As mentioned before, the preferences of trapped economy are directed towards today. Then, the economy at low equilibrium would have less to consume at long run. Transitional dynamics of consumption confirm as well this result. We can understand more in depth this result by looking at consumption rule (8). Since last two terms in (8) are less important for the trapped economy due to higher marginal utility of consumption, hazard adjusted discount level dominates the two level effects discussed in previous sections. Consequently, economy applies a more aggressive management policy and consumes less at long run when it suffers from a trap.

At high equilibrium, the economy adopts a precautionary management policy since the marginal utility of consumption is lower relative to low-equilibrium economy. This means that level effects and growth effect dominate the adjusted hazard rate $\rho + h(S)$, resulting in a higher protection of environment. We observe that a higher steady-state level of consumption for low risk profile at high equilibrium. This can be explained by the fact that the steady-state level of environmental quality exceeds maximum sustainable yield level (MSY)[18]. It is straightforward to remark that accumulating more environmental stock after MSY provides less consumption. However, the lower slope of consumption dynamics for high risk profile economy shows that economy adopts a precautionary behavior.

4.2 The economy with only adaptation policy

The dynamics of accumulation of adaptation shows at which extent the economy is exposed to abrupt event risk. As shown in (16), at steady-state, the economy starts to accumulate more adaptation capital when environmental quality is low. In order to see this, we simulate the model with only adaptation policy. Figure (7) and (8) show the time profile of adaptation capital accumulation and environmental quality for low and high equilibrium cases.

[18] We use a logistic growth function for environmental quality regeneration similar to Ren and Polasky (2014). (See Appendix (6.6)for details.)

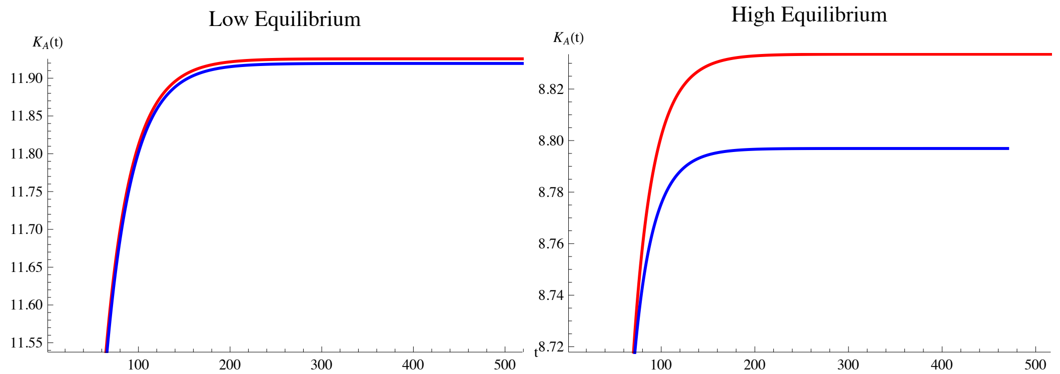

Figure 7: Adaptation capital dynamics at low and high equilibrium

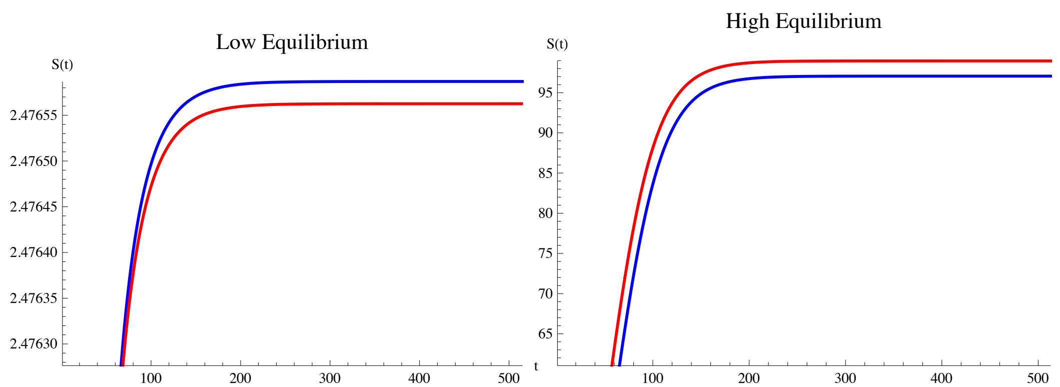

Figure 8: Environmental quality dynamics at low and high equilibrium

Apparently, the low equilibrium economy accumulates much more adaptation capital since it overuses the environment and hence faces a higher abrupt event risk. However, the major concern is that low environmental quality requires more adaptation capital, resulting in a lower environmental quality over time and hence a vicious cycle of "higher adaptation capital - lower environmental quality". Therefore, adaptation capital is likely to trap the economy to low equilibrium if initial state of environmental quality is already low.

Higher risk profile implies an aggressive and precautionary management for trapped and high-equilibrium economy respectively for aforementioned reasons. Adaptation capital increases with higher risk in low-equilibrium economy as expected. However, the increase of adaptation capital for high equilibrium economy may be a counterintuitive result since high risk pushes the economy to be more precautionary. Indeed, the increase of adaptation capital in high equilibrium finds its explanation in an increased marginal benefit of investing in adaptation activity with higher event probability.

Since implementing adaptation capital is likely to cause poverty traps, we argue that adaptation policy should be coupled with an appropriate mitigation policy such that a possible poverty trap can be avoided. For this purpose, we solve the full model with environmental policy in the following subsection and show that it is possible to have an unique equilibrium.

4.3 The economy with full environmental policy

Figure (9) shows optimal time path of adaptation over mitigation investment and resource stock in an unique equilibrium economy. The ratio between optimal adaptation and mitigation investment is higher for a high risk profile economy since the marginal benefit of accumulating adaptation capital is higher due to higher probability of catastrophic event. However, the economy is now able to have a higher environmental quality stock due to mitigation activity, despite the increased ratio of adaptation over mitigation.

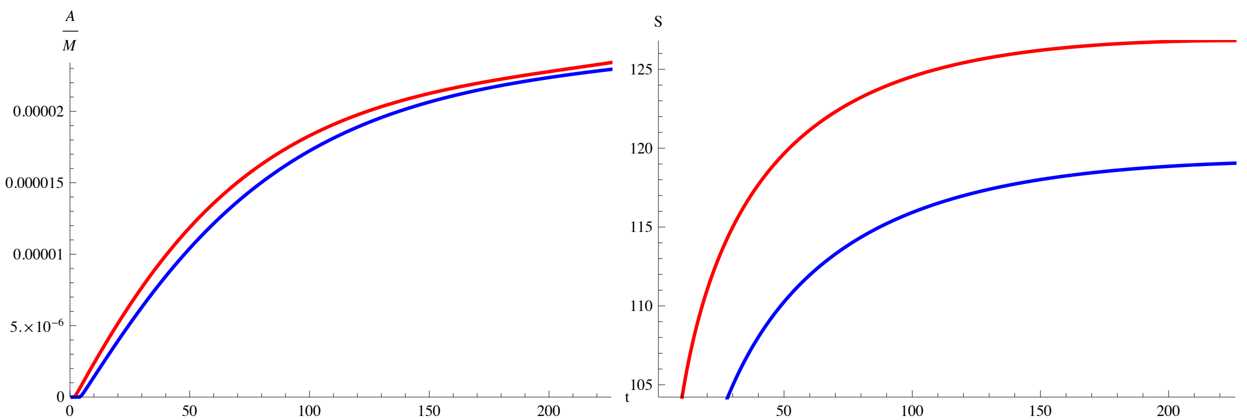

Figure 9: Optimal adaptation vs mitigation and environmental quality dynamics

This result differs from what we have analyzed in the economy implementing only adaptation policy in the sense that economy overuses the environment. Notice from the results above that when the economy implementing only adaptation policy suffers from poverty trap, the resource exploitation policy is more aggressive with higher risk. Nonetheless, the unique equilibrium economy implementing both adaptation and mitigation policy is able to protect environment when it faces a higher risk, by also increasing its adaptation capacity to abrupt events. Since the economy protects the environment by investing in mitigation activity, agents can avoid a high event probability due to which they can not make far sighted decisions and tend to fulfill their basic needs for survival.

5 Conclusion

In this paper, we analyzed the effect of adaptation and mitigation policy on poverty traps in an economy subject to abrupt event risk. The contribution of the study is to offer a new explanation for poverty traps by the abrupt event probability and understand how environmental policy takes an important role to cause or avoid development traps. We believe that this new perspective gives also interesting intuitions to policy makers. Our main results show that adaptation policy can trap an economy to poverty trap while mitigation helps to avoid a poverty trap. We show that a new trade-off appears between adaptation and mitigation concerning their effect on poverty traps, other than the trade-off between these two policies over time mentioned in numerous studies. (see Zemel (2015), Tsur and Zemel (2015), Bréchet et al. (2012)). The fact that adaptation policy could cause a poverty trap does not mean that social planner should stop to invest in adaptation activity. On the contrary, since it is impossible to eliminate completely the hazardous event risk, she should invest in adaptation capital but should couple this policy with mitigation activity to avoid the adverse effects of adaptation policy. This is because mitigation activity weakens the trade-off between present consumption and abrupt event, by improving the environmental quality.

Besides, since there is no available data for adaptation investments in aggregate level, testing the model's results by empirical methods is very challenging for the moment but it is very desirable and also planned in our future research agenda.

6 Appendix

6.1 Derivation of (8).

To solve problem (5)-(1), we reformulate the problem by using Uzawa's transformation (Francis and Kompas (2015a)). This transformation is useful to make a phase diagram analysis, keeping a two dimensional dynamic system in benchmark model. [19]

$$\max_{c} \int_{t}^{\infty} \left[\frac{u(c) - \bar{\psi} h(S)}{\theta(S)} \right] e^{-q} d\theta \tag{21}$$

subject to the budget constraint ;

$$\frac{dS}{dq} = \frac{R(S) - c}{\theta(S)} \tag{22}$$

where $\frac{dq}{\theta(S)} = dt$. The Hamiltonian is

$$\mathcal{H} = \frac{u(c) - \bar{\psi} h(S)}{\theta(S)} + \lambda \left[\frac{R(S) - c}{\theta(S)} \right] \tag{23}$$

The first order conditions are

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{u^{'}(c)}{\theta(S)} - \frac{\lambda}{\theta(S)} = 0 \tag{24}$$

$$\frac{\partial \mathcal{H}}{\partial S} = \lambda - \frac{d\lambda}{dq} = -\left(\frac{\theta^{'}(S)}{\theta^{2}(S)} u(c) - \bar{\psi} h(S) \right) - \frac{\bar{\psi} h^{'}(S)}{\theta(S)} + \frac{\lambda R^{'}(S)}{\theta^{2}(S)} - \lambda \frac{\theta^{'}(S)}{\theta^{2}(S)} [R(S) - c] \tag{25}$$

Multiplying (25) by $\theta(S)$, we convert the problem from virtual time to real time ;

$$\theta(S) \lambda - \frac{d\lambda}{dt} = -\frac{\theta^{'}(S)}{\theta(S)} \left(u(c) - \bar{\psi} h(S) \right) - \bar{\psi} h^{'}(S) + \lambda R^{'}(S) - \frac{\lambda \theta(S)}{\theta'(S)} [R(S) - C] \tag{26}$$

Then, some straightforward calculations yields modified Keynes-Ramsey rule (8).

6.2 Proof of Proposition 1

The first part of the proof starts by analyzing the limits of a function (let this function $G(S)$) that describes the steady state of the economy by a single equation at long run. Then, in the second part, our attention is concentrated on the form of $G(S)$ function and related necessary conditions for multiple steady state.

(a) In some sense, the function $G(S)$ could be considered as the equation $\dot{c} = 0$ as a function of S at steady state equilibrium. Writing down equations $\dot{c} = 0$ and $\dot{S} = 0$;

$$R^{'}(S) - \theta(S) - \frac{\bar{\psi} h^{'}(S)}{u'(c)} - \frac{\theta^{'}(S)}{\theta(S)} \left[\frac{u(c) - \bar{\psi} h(S)}{u'(c)} \right] = 0 \tag{27}$$

$$R(S) - c = 0 \tag{28}$$

[19]Note that same calculations can be obtained in the same manner by a standard resolution method with two state variables.

Plugging equation (28) in (27) yields

$$G(S) = R'(S) - \theta(S) - \frac{\bar{\psi} h'(S)}{u'(R(S))} - \frac{\theta'(S)}{\theta(S)} \left[\frac{u(R(S)) - \bar{\psi} h(S)}{u'(R(S))} \right] \tag{29}$$

Note that we limit our analysis between $S \in [0, \bar{S}]$. We can easily say that the function $G(S)$ starts with a positive value and tends to be negative when S approaches $\bar{S}$. In this framework, $\bar{S}$ is the level of environmental quality level where consumption level is equal to zero. With these information, it is easy to verify $\lim_{S \to 0} G(S) = \infty$ and $\lim_{S \to \bar{S}} G(S) = z < 0$.

(b) In this part, we show the necessary conditions for the existence of multiple steady state, which allow also to represent the function $G(S)$;

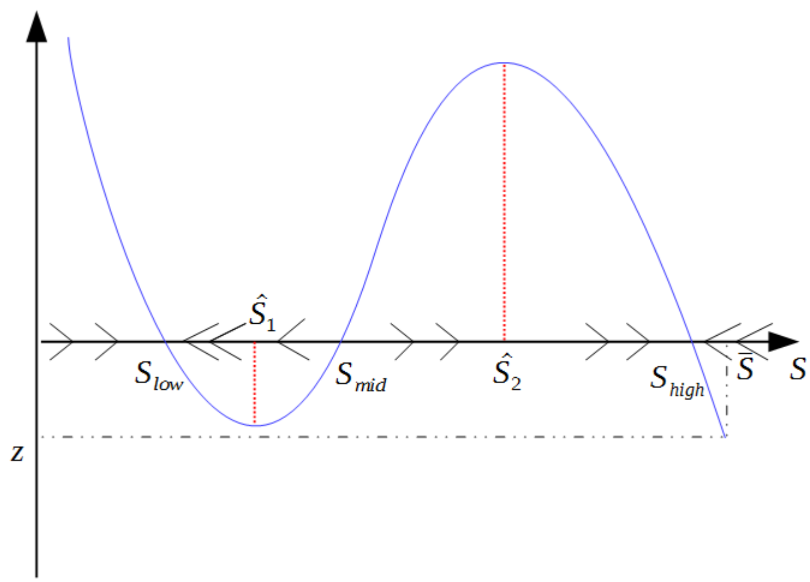

Figure 10: $G(S)$ function with uncertainty

The sufficient condition for the existence of S_{low} is that $\exists S < \widetilde{S}$ [20] which ensures (i) $G'(S) > 0$ and (ii) $G(S) < 0$. Unless the condition (i) $G'(S) > 0$ is satisfied, the function $G(S)$, starting from m may cross x-axis just one more time and converge to z, which results in an unique steady state equilibrium. Additionally, the condition (ii) $G(S) < 0$ is also necessary to ensure that $G(S)$ function crosses the x-axis by S_{low} at least once.

Recall that $\hat{S}_1$ and $\hat{S}_2$ are points of inflection. After these points, $G'(S)$ changes sign. Understanding these critical points is important to precise the directions of arrows for phase diagram analysis on plane (S, c).

The necessary condition for the existence of S_{high} is that $G\left(\widetilde{S}\right) > 0$ for $\exists \widetilde{S} < \bar{S}$. If this condition does not hold, we have a $G(S)$ function not crossing x-axis for the second time and converging directly to z without changing sign. Then, there exists an unique equilibrium. Once condition $G\left(\widetilde{S}\right) > 0$ is satisfied, we

[20] Note that $\widetilde{S} > S_{low}$.

can observe that the function $G(S)$ crosses unambiguously x-axis by S_{mid} after tends to z when S approaches $\bar{S}$. With necessary conditions, we prove the existence of three steady states, one being unstable and two others being stable.

When there does not exist an endogenous occurrence probability, the necessary condition (9) reduces to $R^{''}(S) > 0$, which makes multiple equilibria an impossible outcome. Therefore, the model transforms into a standard neoclassical growth model. This completes the proof.

6.3 Slope of the steady-state curve

By using equation (29) and implicit function theorem, we can find the slope of $\dot{c} = 0$ line.

$$\left.\frac{dc}{dS}\right|_{\dot{c}=0} = \frac{R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\frac{u(R(S)) - \bar{\psi}h(S)}{u^{'}(R(S))} - \frac{\bar{\psi}h^{''}(S)}{u^{'}(R(S))}}{\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{\left(u(c) - \bar{\psi}h(S)\right)u^{''}(c)}{\left(u^{'}(c)\right)^2}\right) - \frac{u^{''}(c)h^{'}(S)\bar{\psi}}{\left(u^{'}(c)\right)^2}} \tag{30}$$

We can remark that the denominator is always negative if [21]

$$\bar{\psi} < \frac{\left(u^{'}(c)\right)^2 - u(c)\,u^{''}(c)}{\rho u^{''}(c)} \tag{31}$$

The sign of necessary condition (9) is crucial in order to determine the sign of $\frac{dc}{dS}\big|_{\dot{c}=0}$. We know that last two terms of necessary condition for multiple equilibria are positive. Since the nominator is quite similar to condition (9), it is easy to notice that nominator is unambiguously negative within zone A and C when $G^{'}(S) < 0$, which makes the slope of $\dot{c} = 0$ line positive at these areas. Within the zone B, since $G^{'}(S) > 0$, we can not determine if nominator is positive or negative.

6.4 Proof of Lemma 1

The differential system describing the economy can be written as follows ;

$$\begin{bmatrix} \dot{c} \\ \dot{S} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{c}}{dc} & \frac{d\dot{c}}{dS} \\ \frac{d\dot{S}}{dc} & \frac{d\dot{S}}{dS} \end{bmatrix}_{\dot{c}=0,\dot{S}=0} \begin{bmatrix} c - c^* \\ S - S^* \end{bmatrix}$$

[21]We verify that this assumption holds in numerical analysis.

$$\frac{d\dot{c}}{dc} = \theta(S) - R'(S)$$

$$-\frac{u'(c)}{u''(c)}\left[R''(S) - \theta'(S) - \frac{\bar{\psi}h''(S)}{u'(c)} - \left(\frac{\theta'(S)}{\theta(S)} - \frac{\left(\theta'(S)\right)^2}{(\theta(S))^2}\right)\left[\frac{u(c) - \bar{\psi}h(S)}{u'(c)}\right] - \frac{\theta'(S)}{\theta(S)}R'(S)\right]$$

$$\frac{d\dot{S}}{dc} = -1$$

$$\frac{d\dot{S}}{dS} = R'(S)$$

We know that for a saddle-stable path system, it is necessary to have one positive and one negative eigenvalue, denoted $\mu_{1,2}$. As the $Tr(J) = \mu_1 + \mu_2$ and $Det(J) = \mu_1\mu_2$. It is sufficient to show that $Tr(J) > 0$ and $Det(J) < 0$. It is easy to see that $Tr(J) = \theta(S) > 0$ and with arranging the terms for the determinant, we can see that determinant reduces to the multiple steady state condition $G(S)$. We conclude that $Det(J)$ is negative if

$$G'(S) = R''(S) - \theta'(S) - \left[\frac{\theta''(S)\theta(S) - \left(\theta'(S)\right)^2}{(\theta(S))^2}\right]\frac{\left(u(R(S)) - \bar{\psi}h(S)\right)}{u'(R(S))} - \frac{\bar{\psi}h''(S)}{u'(R(S))} + \frac{\bar{\psi}u''(R(S))}{(u'(R(S)))^2} \tag{32}$$

$$-\left[\frac{\theta'(S)}{\theta(S)}\left(1 - \frac{\left(u(c) - \bar{\psi}h(S)\right)u''(c)}{(u'(c))^2}\right)\right]\left[R'(S)\right] < 0$$

Complex dynamics arise if $(Tr(J))^2 - 4Det(J) < 0$.

$$(\theta(S))^2 < 4\frac{u'(c)}{u''(c)}\left[R''(S) - \theta'(S) - \frac{\bar{\psi}h''(S)}{u'(c)} - \left(\frac{\theta'(S)}{\theta(S)} - \frac{\left(\theta'(S)\right)^2}{(\theta(S))^2}\right)\left[\frac{u(c) - \bar{\psi}h(S)}{u'(c)}\right] - \frac{\theta'(S)}{\theta(S)}R'(S)\right] \tag{33}$$

As $Det(J)$ is shown to be negative for low and high steady states, this prevents these to steady states to have complex dynamics. However, for the middle steady state there is a possibility to have complex dynamics arises if the condition above is ensured.

6.5 Necessary conditions for an optimal policy in the model with environmental policy

In this part of the appendix, we present the first order conditions associated with the two-states and three-controls dynamic optimization problem (14). The current-value Hamiltonian is

$$\mathcal{H} = \frac{u(c) - Q_2(M) - Q_1(A) - \psi(K_A)h(S)}{\theta(S)} + \lambda\left[\frac{R(S) - c + \Gamma(M)}{\theta(S)}\right] + \mu\left[\frac{A - \delta K_A}{\theta(S)}\right] \tag{34}$$

where λ and μ are the current-value co-state variables for S and K_A, respectively. First order conditions for an internal optimal solution give

$$\frac{\partial \mathcal{H}}{\partial c} = \frac{u^{'}(c)}{\theta(S)} - \frac{\lambda}{\theta(S)} = 0 \tag{35}$$

$$\frac{\partial \mathcal{H}}{\partial A} = -\frac{Q_1^{'}(A)}{\theta(S)} + \frac{\mu}{\theta(S)} = 0 \tag{36}$$

$$\frac{\partial \mathcal{H}}{\partial M} = -\frac{Q_2^{'}(M)}{\theta(S)} + \frac{\lambda \Gamma^{'}(M)}{\theta(S)} = 0 \tag{37}$$

$$\dot{c} = -\frac{u^{'}(c)}{u^{''}(c)}\left[R^{'}(S) - \theta(S) - \frac{\psi(K_A)h^{'}(S)}{u^{'}(c)} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(c) - \psi(K_A)h(S) - Q_2(M) - Q_1(A)}{u^{'}(c)} + \frac{Q_1^{'}(A)}{u^{'}(c)}\dot{K}_A + \dot{S}\right]\right] \tag{38}$$

$$\dot{A} = \frac{Q_1^{'}(A)}{Q_1^{''}(A)}\left[\theta(S) + \delta + \frac{\psi^{'}(K_A)h(S)}{Q_1^{'}(A)}\right] \tag{39}$$

By using equations (12), (37), (38) and (39) at steady-state, we can write $G(S)$ in the model with environmental policy

$$G(S) = R^{'}(S) - \theta(S) - \frac{\psi(K_A(S))h^{'}(S)}{u^{'}(c(S, M(S)))} - \frac{\theta^{'}(S)}{\theta(S)}\left[\frac{u(c(S, M(S))) - \psi(K_A(S))h(S) - Q_2(M(S)) - Q_1(\delta K_A(S))}{u^{'}(c(S, M(S)))}\right] \tag{40}$$

where one should write K_A, M and c as a function of environmental quality S. For the sake of notation simplicity, we prefer to not write each variable as a function of S in the remainder of the paper. The necessary condition for multiple equilibria in full model yields

$$G_3^{'}(S) = X_3(S) + \underbrace{\left[\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S) - \theta(S)) + \delta Q_1^{'}(\delta K_A)\right]\right]\frac{dK_A}{dS}}_{>0}$$

$$-\frac{\theta^{'}(S)}{\theta(S)}\underbrace{\left[\left(\frac{u^{''}(c)(u(c) - \psi(K_A)h(S) - Q_1(\delta K_A) - P_M M)}{(u^{'}(c))^2}\right) - \frac{\psi(K_A)u^{''}(c)h^{'}(S)}{(u^{'}(c))^2} - \frac{P_M}{u^{'}(c)\Gamma^{'}(M)}\right]\Gamma^{'}(M(S))\frac{dM}{dS}}_{<0} > 0 \tag{41}$$

where

$$X_3(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\left[\frac{u(c) - \psi(K_A)h(S) - Q_1(A) - P_M M}{u^{'}(c)}\right]$$

$$-\left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{(u(c) - \psi(K_A)h(S) - Q_1(A) - P_M M)u^{''}(c)}{(u^{'}(c))^2} - \frac{\psi(K_A)h^{'}(S)}{u^{'}(c)R^{'}(S)}\right)\right]\left[R^{'}(S)\right] \tag{42}$$

6.5.1 An economy with only adaptation policy

When social planner implements only adaptation policy, we set $M = 0$ and solve the model with only adaptation policy. For the sake of space, we don't present calculations in appendix. The necessary condition becomes :

$$G_1^{'}(S) = X_1(S) + \underbrace{\left[\frac{\theta^{'}(S)}{\theta(S)}\left[\frac{\psi^{'}(K_A)}{u^{'}(c)}(h(S)-\theta(S)) + \delta Q_1^{'}(\delta K_A)\right]\right]\frac{dK_A}{dS}}_{Z_1>0}$$

where

$$X_1(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\left[\frac{u(c) - \psi(K_A)h(S) - Q_1(A)}{u^{'}(c)}\right]$$
$$- \left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{(u(c) - \psi(K_A)h(S) - Q(A))u^{''}(c)}{(u^{'}(c))^2} - \frac{\psi(K_A)h^{'}(S)}{u^{'}(c)R^{'}(S)}\right)\right]\left[R^{'}(S)\right] \tag{43}$$

6.5.2 An economy with only mitigation policy

When social planner implements only mitigation policy, we set $A = K_A = 0$ and solve the model with only mitigation policy. The necessary condition becomes :

$$G_2^{'}(S) = X_2(S) - \frac{\theta^{'}(S)}{\theta(S)}\underbrace{\left[\left(\frac{u^{''}(c)\left(u(c) - \bar{\psi}h(S) - P_M M\right)}{(u^{'}(c))^2}\right) - \frac{\bar{\psi}u^{''}(c)h^{'}(S)}{(u^{'}(c))^2} - \frac{P_M}{u^{'}(c)\Gamma^{'}(M)}\right]\Gamma^{'}(M(S))\frac{dM}{dS}}_{Z_2<0} > 0 \tag{44}$$

where

$$X_2(S) = R^{''}(S) - \theta^{'}(S) - \left[\frac{\theta^{''}(S)\theta(S) - \left(\theta^{'}(S)\right)^2}{(\theta(S))^2}\right]\left[\frac{u(c) - \bar{\psi}h(S) - P_M M}{u^{'}(c)}\right]$$
$$- \left[\frac{\theta^{'}(S)}{\theta(S)}\left(1 - \frac{\left(u(c) - \bar{\psi}h(S) - P_M M\right)u^{''}(c)}{(u^{'}(c))^2} - \frac{\bar{\psi}h^{'}(S)}{u^{'}(c)R^{'}(S)}\right)\right]\left[R^{'}(S)\right] \tag{45}$$

The term Z_2 is negative if the following condition on constant unit cost of mitigation P_M is ensured[22].

$$P_M < \frac{\frac{\left(u(c)-\bar{\psi}h(S)\right)u^{''}(c)}{\left(u^{'}(c)\right)^2} - \frac{\bar{\psi}u^{''}(c)h^{'}(S)}{\left(u^{'}(c)\right)^2}}{\frac{Mu^{''}(c)}{\left(u^{'}(c)\right)^2} + \frac{1}{u^{'}(c)\Gamma^{'}(M)}}$$

6.6 Functional forms and parameter values

[22] We verify that this condition holds for all numerical exercises.

Natural Regeneration Function : $R(S) = S + gS\left(1 - \frac{S}{\bar{S}}\right)$	Utility function : $u(c) = \frac{c^{1-\sigma} - c_{min}^{1-\sigma}}{1-\sigma}$
Source : Ren and Polasky (2014)	Source : Bommier et al. (2015)
$\bar{S}$ Carrying capacity of environment g Intrinsic growth rate of the resource stock	c_{min} Post-value consumption σ Degree of relative risk aversion
Penalty function : $\psi(K_A) = \bar{\psi}\left(\omega + (1-\omega)e^{-\gamma K_A}\right)$	Hazard Function : $h(S) = \frac{2h}{1+exp[\eta(s/s^*-1)]}$
Source : Bréchet et al. (2012)	Source : Ren and Polasky (2014)
$\bar{\psi}$ Penalty rate without adaptation policy ω Lower bound of penalty when $\psi(\infty)$ γ Elasticity of adaptation w.r.t to penalty rate	$\bar{h}$ Upper bound for hazard rate η Endogeneity level of catastrophic event s^* Risk-free steady state of resource stock
Mitigation function : $\Gamma(M) = M^\alpha$	Cost of adaptation investment : $Q_1(A) = \phi_A \frac{A^2}{2}$
Source : Kama and Pommeret (2016)	Source : Quadratic cost function
α Elasticity of mitigation activity	ϕ_A Parameter for the change of marginal cost of adaptation

Table 1: Functional forms

In order to make comparison between benchmark model and model with only adaptation policy, we use the following parameter values :

Parameters	Benchmark model	Model with only adaptation policy
σ	1.05	1.05
c_{min}	1	1
ρ	0.025	0.025
g	0.05	0.05
$\bar{S}$	52	52
η	4	4
$\bar{h}$	0.5	0.5
$\bar{\psi}$	10	10
ω	-	0.185
γ	-	0.6
δ	-	0.065
ϕ_A	-	0.5

Table 2: Parameter values for benchmark model and model with only adaptation policy

We use the following parameter values to compare benchmark model to model with only mitigation policy :

Parameters	Benchmark model	Model with only mitigation policy
σ	1.5	1.5
c_{min}	1	1
ρ	0.025	0.025
g	0.05	0.05
$\bar{S}$	51.25	51.25
η	5	5
$\bar{h}$	0.5	0.5
$\bar{\psi}$	10	10
α	-	0.75
P_M	-	0.0005

Table 3: Parameter values for benchmark model and model with only mitigation policy

We use the following parameter values for the full model :

Parameters	Full Model
σ	1.5
c_{min}	1
ρ	0.025
g	0.05
$\bar{S}$	51.25
η	5
h	0.5
$\bar{\psi}$	10
α	0.75
P_M	0.0005
ω	0.9
γ	0.6
δ	0.065
ϕ_A	0.5

Table 4: Parameter values for full model

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